The Thirtieth Annual
Eastern Shore High School Mathematics Competition

November 7, 2013

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

There is no penalty for incorrect answers.

The use of a calculator is not permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. For which real values of x is $x^8 \quad 8x^4 + 16 = 0$?

   a. $x = \sqrt{2}$ b. $x = 2$ c. $x = 4$ d. $x = \sqrt{2}; \sqrt{2}$ e. $x = \quad 2; 2$

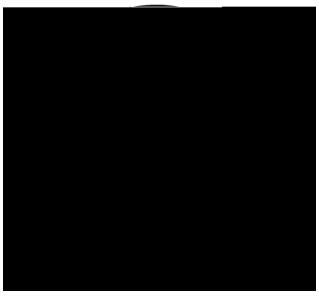
2. Suppose $f(x) = ax^2 + bx + c$ is a real-valued function with two distinct real zeros. Which of the following choices for $g(x)$ guarantees that $g(x)$ also has two distinct real zeros?

   a. $x^2 + bx + 4ac$ b. $2ax^2 + bx + c$ c. $ax^2 + 2bx + c$ d. $cx^2 + bx + a$ e. $x^2 \quad bx + 2ac$

3. The solution to the inequality $\sqrt{x^2 \quad x \quad 12} < x$ is

   a. $x > 0$
   b. $x \quad 4$
   c. $x > \quad 12$
   d. $0 < x < 4$
   e. $\quad 3 < x < 4$

4. O is the center of the circle pictured below. $\overline{OH} = 10$ units and $ABCD$ is a square inscribed in the circle. What is the length of the minor arc AB?



   a. $\sqrt{2}$ units b. $3\sqrt{2}$ units c. $5\sqrt{2}$ units d. $10\sqrt{2}$ units e. $20\sqrt{2}$ units

5. In the television series Pretty Little Liars, several high school girls are consistently terrorized by a group of unknown evildoers called Team A. The girls are desperate to discover the identities of the members of Team A. After intensive investigation, one of the girls, Hanna, has deduced that the only possible members of Team A are Toby, Mona, Alison, Spencer, and CeCe. In addition, Hanna knows that Team A consists of only two people. Assuming that Toby, Mona, Alison, Spencer, and CeCe are each equally likely to belong to Team A, what is the probability that Spencer is a member of Team A?

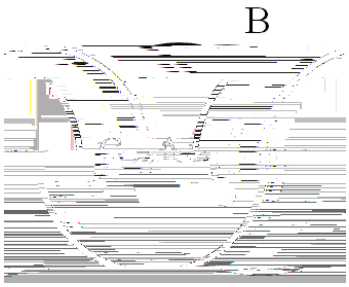
   a. $\frac{1}{20}$ b. $\frac{1}{5}$ c. $\frac{1}{9}$ d. $\frac{1}{3}$ e. $\frac{2}{5}$

6. How many distinct prime factors does 2013 have?

   a. 1 b. 2 c. 3 d. 4 e. More than 4

7. In the sleepy town of Mystic Falls live four vampires and 10,000 humans. Elena, a new vampire, cannot control her insatiable urge to turn humans into vampires. As a result, the number of humans in Mystic Falls is decreasing while the population of vampires is increasing. At any time t the number of humans is given by the formula

12. $\overline{AC}$ is a diameter of the circle shown below and $\overline{BD}$ is perpendicular to $\overline{AC}$.



Then, $(AD)(DC)$ is equal to

a. $(AB)^2$ b. $(BC)^2$ c. $(BD)^2$ d. $(AC)^2$ $(DC)^2$ e. Answer Not Shown

13. The sum of the values in the solution set of $\tan^2 x = \sin 2x$ over the interval $0 \quad x < 2$ is

a. $\frac{}{2}$ b. $\frac{3}{4}$ c. d. $\frac{3}{2}$ e. $\frac{5}{2}$

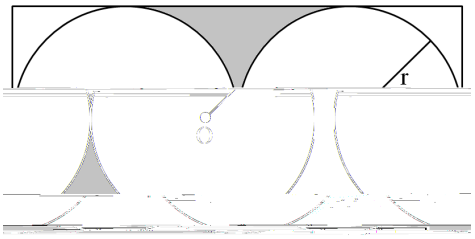
14. Among 300 high school students selected from various high schools on the Eastern shore, the frequencies by gender and attedance at the High School Math Contest (HSMC) are given in the table below.

	Male	Female
Attended HSMC	65	50
Did Not Attend HSMC	100	85
Total	165	135

If one of these 300 students is selected at random, what is the probability that the student attended the HSMC and is female?

a. $\frac{135}{300}$ b. $\frac{50}{300}$ c. $\frac{115}{300}$ d. $\frac{50}{135}$ e. $\frac{50}{115}$

15. In the gure below, r is the length of the radius of both circles. Each circle is tangent to the rectangle on three of the rectangle's sides. The circles are externally tangent. What is the area of the shaded region?



a. $r^2(2^{p}\overline{3} \quad)$ b. $r^2(3^{p}\overline{2} \quad)$ c. $r^2(4 \quad)$ d. $r^2(5 \quad)$ e. $2r^2(4 \quad)$

16. The following set of points can be found on the graph of $f(x \quad 2)$:

    $f(0;4);(1;1);(2;0)g$

    Which of the following functions could be $f(x)$?

    a. $jx \quad 2j$ b. $jx + 2j$ c. jxj d. x^2 e. $(x + 2)^2$

17. Which of the following vertices belong to a triangle that is similar to a right triangle with side lengths 3, 4, and 5?

    a. (0,0), (4,0), (4,5) b. (0,1), (6,9), (6,1) c. (0,0), (6,0), (6,9) d. (0,0), (3,4), (5, 0) e. (0,0), (3,0), (3,5)

18. De ne a sequence a_n such that the rst term of the sequence, a_1, is 1 and the rest of the sequence is generated using the rule

    $$a_n = 10^{2 \quad 2n} \text{ for } n \quad 2$$

    The rst three terms of a_n are 1, $10^{\ 2}$, and $10^{\ 4}$. What is the sum of all the terms in this sequence?

    a. $0:\overline{01}$ b. $0:\overline{1}$ c. 1 d. $1:\overline{01}$ e. $1:\overline{1}$

19. Ruby recently started keeping a journal. She wrote one sentence on the rst day, then two sentences on the second day, then three sentences on the third day, and so on. If she continues in the same way, increasing the lengths of her